The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: November 20, 2018 Course: EE 313 Evans

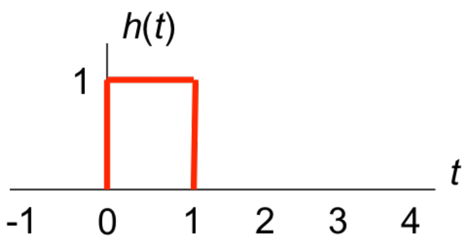
Name: ____
Last, First

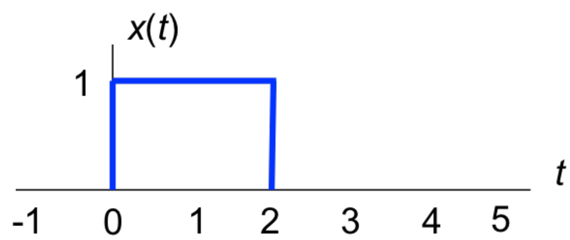
- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computing system(s).***
- Please power down all cell phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	*Point Value*	*Your score*	*Topic*
1	18		Continuous-Time Convolution
2	18		Discrete-Time Convolution
3	20		Discrete-Time First-Order System
4	26		Discrete-Time Second-Order System
5	18		System Properties
Total	100		

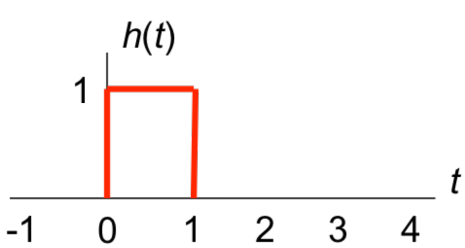
Problem 2.1 *Continuous-Time Convolution.* 18 points.

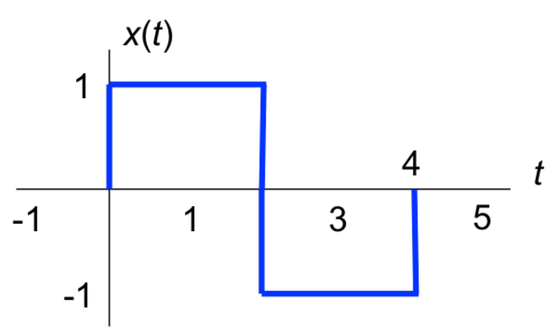
(a) Plot $y(t) = h(t) * x(t)$ using the rectangular pulse signals below. *9 points.*





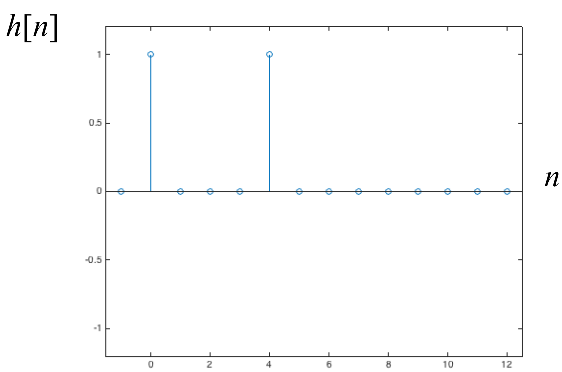
(b) Plot $y(t) = h(t) * x(t)$ using the signals below. *9 points*

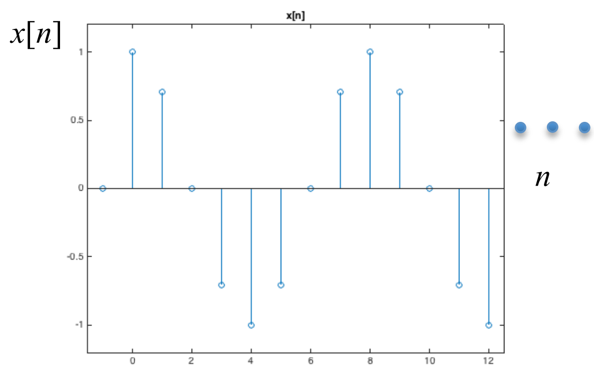




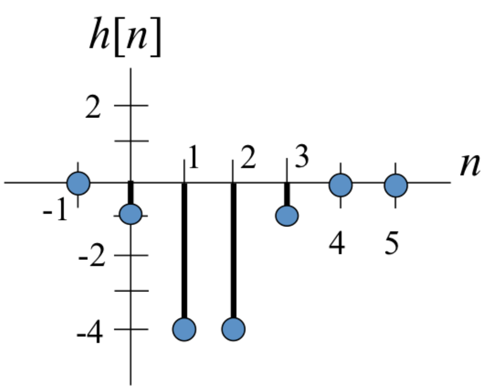
Problem 2.2 *Discrete-Time Convolution*. 18 points.

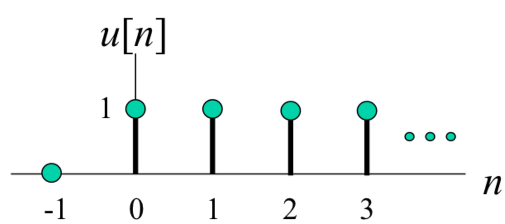
(a) Plot $y[n] = h[n] * x[n]$ where $h[n] = \delta[n] + \delta[n-4]$ and $x[n] = \cos(\pi\, n / 4)\, u[n]$ such that $u[n]$ is the unit step function. Below, $h[n]$ and $x[n]$ are plotted for $-1 \le n \le 12$, but please keep in mind $x[n]$ is a cosine over $0 \le n < \infty$. *9 points.*





(b) Plot $y[n] = h[n] * u[n]$ using the signals below. Here, $h[n] = -\delta[n] - 4\,\delta[n-1] - 4\,\delta[n-2] - \delta[n-3]$ and is plotted below. Also, $u[n]$ is the unit step function and is plotted below. *9 points*.





Problem 2.3. *Discrete-Time First-Order LTI IIR System.* 20 points**.**

Consider a causal discrete-time first-order linear time-invariant (LTI) system with input $x[n]$ and output $y[n]$ governed by the following input-output relationship

$$y[n] - a\, y[n\text{-}1] = x[n] - b\, x[n\text{-}1]$$

for real-valued constants a and b where $|a| < 1$. Input and output signals are observed for $n \geq 0$.

(a) What are the initial conditions? What should their values be? Why? *2 points.*

(b) Derive the transfer function in the z-domain including the region of convergence. *3 points*.

(c) Give a formula for the frequency response. *3 points.*

(d) Give values of a and b for a lowpass filter. Both values of a and b should be close to 1 in absolute value. Make sure that the filter is bounded-input bounded-output stable. Justify your choices. *6 points.*

(e) Give values of a and b for an allpass filter. Both values of a and b should be close to 1 in absolute value. Make sure that the filter is bounded-input bounded-output stable. Justify your choices. *6 points*

Problem 2.4 *Discrete-Time Second-Order LTI System.* 26 points.

The transfer function in the z-domain for a causal discrete-time second-order linear time-invariant (LTI) system with zeros z_0 and z_1 and poles p_0 and p_1 is given by

$$H(z) = \frac{(z - z_0)(z - z_1)}{(z - p_0)(z - p_1)}$$

We can write the transfer function in negative powers of z as follows:

$$H(z) = \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})} = \frac{1 - (z_0 + z_1)z^{-1} + z_0 z_1 z^{-2}}{1 - (p_0 + p_1)z^{-1} + p_0 p_1 z^{-2}} = \frac{1 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

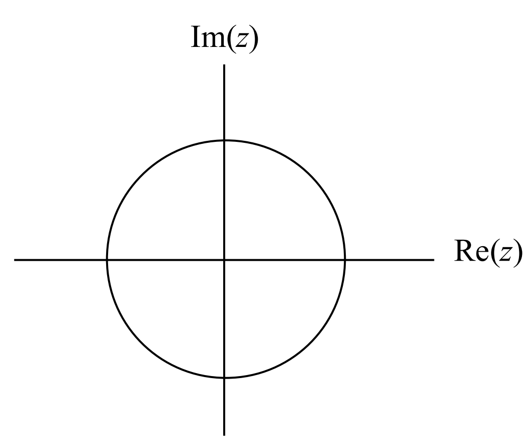
In the discrete-time domain, the input and output signals are observed for $n \geq 0$.

(a) What is the region of convergence for $H(z)$? *3 points*.

(b) Derive the difference equation that relates input $x[n]$ and output $y[n]$ in the discrete-time domain. *6 points.*

(c) What are the initial conditions? To what values should the initial conditions be set? *3 points*.

(d) Choose the values of the two poles and two zeros to design a bandpass filter to pass the fourth octave on the Western music scale and attenuate the other octaves as much as possible. In the fourth octave, the lowest note is at 262 Hz, the highest note is at 494 Hz, and the center frequency is at 378 Hz. Use a sampling rate of 8000 Hz. Your filter design must be bounded-input bounded-output stable. Please plot the poles and zeros on the pole-zero diagram below. *14 points*.



Problem 2.5. *System Properties.* 18 points.

Each of the following discrete-time systems has input $x[n]$ and output $y[n]$.

Determine if each system is linear or nonlinear, time-invariant or time-varying, and bounded-input bounded-output (BIBO) stable.

You must either prove that the system property holds in the case of linearity, time-invariance, or BIBO stability, or provide a counter-example that the property does not hold. Providing an answer without any justification will earn 0 points.

Please note that $x[n]$ and $y[n]$ might be complex-valued.

Part	***System Name***	***System Formula***	***Linear?***	***Time-Invariant?***	***BIBO stable?***
(a)	Squaring System	$y[n] = x^2[n]$			
(b)	Real part	$y[n] = \text{Re}\{ x[n] \}$			
(c)	Running Summation	$y[n] = y[n-1] + x[n]$ for $n \geq 0$			

(a) Squaring system: $y[n] = x^2[n]$. *6 points*.

(b) Real part: $y[n] = \text{Re}\{ x[n] \}$. *6 points*.

(c) Running summation: $y[n] = y[n-1] + x[n]$ for $n \geq 0$. *6 points*.

```
% MATLAB Code

% Code for problem 2.2(a)
n = -1:12;
h = zeros(1,14);
h(2) = 1;
h(6) = 1;
stem(n, h);
ylim( [-1.2 1.2] );
xlim( [-1.5 12.5] );

n = -1:12;
x = cos(pi*n/4);
x(1) = 0;
stem(n, x);
ylim( [-1.2 1.2] );
xlim( [-1.5 12.5] );
title( 'x[n]' );
```